Supplemental Material:
Recovering Transparent Shape from Time-of-Flight Distortion

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Appendices

In this supplemental material, we discuss in detail of our method that is omitted in the main text due to the page limit.

A. Determining s from hypothesized t using ToF length

The back surface point $\mathbf{b}$ has two equality constraints among measurements and parameters. The first equality is that $\mathbf{b}$ is on the reference ray as

$$\mathbf{b} = \mathbf{r}_1 - s\mathbf{v}_3. \tag{1}$$

The other is that the optical length satisfies the ToF distortion model as

$$t + \nu\left|\mathbf{b} - \mathbf{f}\right| + \left|\mathbf{r}_1 - \mathbf{b}\right| = l_{ToF}. \tag{2}$$

The solution of the back surface point is the crossing point of the line (Eq. (1)) and quadratic surface (Eq. (2)) as shown in Fig. 1.

Squaring Eq. (2) after substitution of $\mathbf{b}$ for Eq. (1) and transposition, we get

$$\nu^2\left|\mathbf{r}_1 - s\mathbf{v}_3 - \mathbf{f}\right|^2 = \left(l_{ToF} - t - s\right)^2. \tag{3}$$

Organizing for s,

$$s^2(\nu^2 - 1) + 2s\left(l_{ToF} - t - \nu^2\left(\mathbf{r}_1 - \mathbf{f}\right)^T\mathbf{v}_3\right) + \nu^2\left|\mathbf{r}_1 - \mathbf{f}\right|^2 - \left(l_{ToF} - t\right)^2 = 0. \tag{4}$$

Finally, from the quadratic formula, we get

$$s = \frac{-h \pm \sqrt{h^2 - gi}}{g}, \tag{5}$$

where g, h, and i are auxiliary variable such that

$$\begin{cases} g &= \nu^2 - 1 \\ h &= l_{ToF} - t - \nu^2\left(\mathbf{r}_1 - \mathbf{f}\right)^T\mathbf{v}_3 \\ i &= \nu^2\left|\mathbf{r}_1 - \mathbf{f}\right|^2 - \left(l_{ToF} - t\right)^2. \end{cases} \tag{6}$$

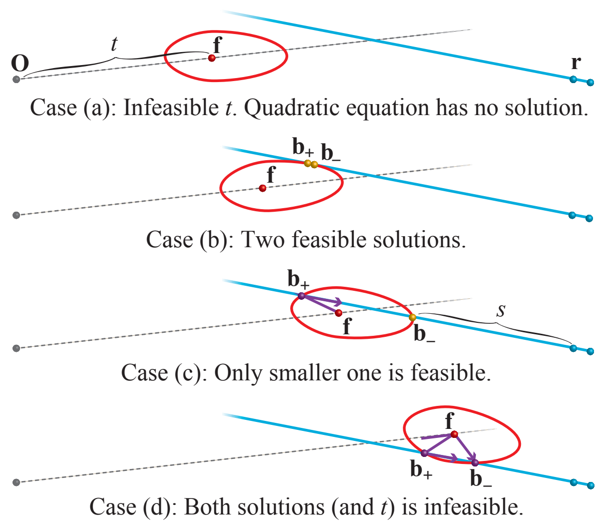

Figure 1: Solution of the back surface point is a crossing point of reference ray (Eq. (1)) illustrated as blue line and quadratic surface (Eq. (2)) illustrated as red circle. (a) If t is too small, there is no intersection hence t is infeasible. (b) Two feasible solutions exist. (c) There are two solutions, but large one ($\mathbf{b}_+$) breaks total reflection condition (purple path). (d) If t is too large, both solutions break total reflection condition or they do not cross hence t is infeasible.

There are some cases based on the relationship between the quadratic surface and reference ray. The first case is that the reference ray does not cross the quadratic surface as shown in Fig. 1(a) because the hypothesized t is too small. In this case, $h^2 - gi < 0$ hence t is infeasible. The second case is that the reference ray is a tangent of the surface or similar situation as shown in (b). In this case, two solution $\mathbf{b}_-$ and $\mathbf{b}_+$ are both feasible. For the continuity of the following third case, we choose smaller s in our implementation. The third case, as shown in (c), is that one of the solution ($\mathbf{b}_+$) is infeasible because the light path from

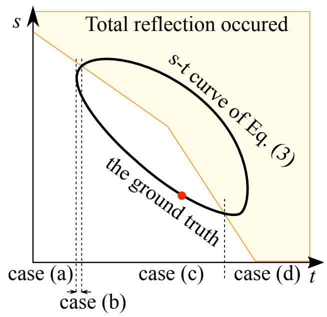

Figure 2: Example of s-t solution space. Solution is on a quadratic curve of Eq. (3) and the red point is the ground-truth. Candidate light paths in the orange region does not satisfy total reflection condition.

$\mathbf{f} \rightarrow \mathbf{b}_+ \rightarrow \mathbf{r}_1$ does not satisfy total reflection condition

$$\mathbf{v_{in}}^T \mathbf{v_{out}} > \cos\theta_c, \tag{7}$$

where $\theta_c = \sin^{-1}(1/\nu)$ is the critical angle. In this case, only smaller s (*i.e.*, $\mathbf{b}_-$) is feasible. The last case is that the hypothesized depth t is too large as shown in (d). In this case, the smaller s is also infeasible because the light path $\mathbf{O} \rightarrow \mathbf{f} \rightarrow \mathbf{b}$ does not satisfy total reflection condition, where $\mathbf{O}$ is the camera position. Furthermore, it could be no solution ($h^2 - gi < 0$). Figure 2 shows an example of s-t solution space. Orange region shows where light paths break total reflection condition, and we can see smaller s is the solution in most cases. This assumption breaks down only if the back refraction is near the total reflection limit, which can appear at the edge of the object.

B. Normal vector from refractive path

The definition of Snell's law in a cross product form is

$$\mathbf{v}_1 \times \mathbf{n} = \nu \mathbf{v}_2 \times \mathbf{n}, \tag{8}$$

where $\mathbf{v}_1$ is an incident ray, $\mathbf{v}_2$ is its refractive ray, and $\mathbf{n}$ is the normal of the surface. By transposition, we can obtain the surface normal direction as

$$\mathbf{n} \propto \nu \mathbf{v}_2 - \mathbf{v}_1. \tag{9}$$

By normalization, we can obtain the surface normal vector.

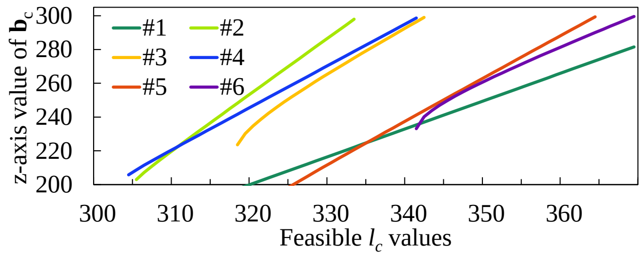

Figure 3: The plot of f_c with respect to l_c. Samples are picked from various simulation objects. They are monotonically increasing in feasible l_c, and look like affine.

C. Convexity of l-subproblem

The value of z axis of the back point f_c can be represented as

$$f_c(l_c) = r_{1,c,z} - s(\hat{t}_c, l_c) v_{3,c,z}, \tag{10}$$

where $r_{1,c,z}$ and $v_{3,c,z}$ are z-axis value of $\mathbf{r}_1$ and $\mathbf{v}_3$ at pixel c, respectively. Using the function f_c, we can rewrite **l**-subproblem (Eq. 11 in the main text) as

$$\begin{aligned}\underset{\mathbf{l}}{\operatorname{argmin}} \sum_{c \in C} \| l_c - l_{ToF}(c) \|_2^2 + \\ \lambda_3'' \sum_{j,k \in N} \| f_j(l_j) - f_k(l_k) \|_H . \end{aligned} \tag{11}$$

If the function f_c is an affine function, the optimization problem can be rewritten into a variant of total variation denoising problem, which is known as a convex problem [1]. In our problem, f_c is not an affine function but we have observed that it has similar properties, which is monotonic increase and small magnitude of second derivative. Figure 3 shows example plots of the function f_c in feasible set of l_c, and they are similar to the affine function. If the light path candidate is near the total reflection condition, this affinity breaks down as shown in #3 and #6. In practice, we assume our objective function should have similar property; the global minimum can be obtained.

References

[1] M. A. Little and N. S. Jones. Sparse Bayesian Step-filtering for High-throughput Analysis of Molecular Machine Dynamics. In *IEEE International Conference on Acoustics, Speech, and Signal Processing, ICASSP*, pages 4162–4165. IEEE, 2010. 2